

Dynamical ultrametricity in the critical trap model

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We show that the trap model at its critical temperature presents dynamical ultrametricity in the sense of Cugliandolo and Kurchan [1]. We use the explicit analytic solution of this model to discuss several issues that arise in the context of mean-field glassy dynamics, such as the scaling form of the correlation function, and the finite time (or finite forcing) corrections to ultrametricity, that are found to decay only logarithmically with the associated time scale, as well as the fluctuation dissipation ratio. We also argue that in the multilevel trap model, the short time dynamics is dominated by the level which is at its critical temperature, so that dynamical ultrametricity should hold in the whole glassy temperature range. We revisit some experimental data on spin-glasses in light of these results.

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I. INTRODUCTION

Notable theoretical progress in our understanding of the ubiquitous *aging* phenomena in glassy systems was made possible by the recognition that the discussion of correlation and response functions requires *two* times: the waiting time t_w and the total elapsed time $t_w + t$. This appears very clearly in the framework of mean-field spin glass models, domain growth (coarsening models), or more phenomenological trap models [2]. Although the basic phenomenology of all these models are rather similar, the underlying physical picture is completely different. For example, aging in domain growth models is associated to the growth of a coherence length. In mean field or trap models, space is absent and aging is related to the structure of the energy landscape, but here again the intuition is completely different. In mean field models, the system never reaches the bottom of an energy valley and there are no activated processes involved in the dynamics. Rather, the dynamics slows down because saddles with less and less ‘descending’ directions are visited as the system ages. Conversely, in the trap model, activation is the basic ingredient of the model, and aging is associated to the fact that deeper and deeper valleys are reached as the system evolves. Dynamics in the latter case is fundamentally *intermittent*: either nothing moves, or there is a jump between two traps. This must be contrasted with mean field dynamics which is continuous in time. However, mean field and trap dynamics can be shown to correspond to two successive time regimes in a particular class of models [3].

In spite of these important differences, many predictions are common to the latter two pictures, such as:

- A short time singularity of the response function in the aging regime, which leads to an (aging) low frequency noise.
- Non trivial violations of the Fluctuation Dissipation Theorem (FDT), first pointed out within mean

field models, but that also exists in trap models.

- The possibility of rejuvenation and memory, which involves the existence of different degrees of freedom with different time scales.

Furthermore, a certain class of mean field models (that includes the Sherrington-Kirkpatrick model) has been conjectured to possess ultrametric dynamical properties, that very precisely reflect and encode the ultrametric nature of the static solution. This dynamical ultrametricity is associated to an infinite number of time scales (which diverges with the age of the system), in the following sense: if $C(t_2, t_1)$ is the correlation function between times $t_1 < t_2$, then in the limit of large times:

$$C(t_3, t_1) = \min(C(t_2, t_1), C(t_3, t_2)), \quad \forall t_2 \in [t_1, t_3]. \quad (1)$$

This means that either t_2 is close enough to t_3 , and then no further dynamics takes place between t_2 and t_3 , or t_2 is close enough to t_1 but then the age of the system hardly changes between t_1 and t_2 . This property of the correlation function has been shown to hold for the aging solution of the dynamical (Mode Coupling) equations describing the dynamics of ‘continuous’ spin-glasses. This property is furthermore invariant under reparametrizations of time, where $t \rightarrow h(t)$ with an arbitrary monotonous function $h(t)$. Testing whether or not dynamical ultrametricity also holds in realistic disordered systems is made difficult because this property is only expected in the limit of asymptotically large times, and corrections are expected on finite times. How large are these corrections?

In this paper, we show that exact ultrametricity holds *at* the critical point of the single-level trap model (or random energy model). We give the explicit form of the correlation function and discuss finite time (or finite forcing) corrections. Note that in this single-level trap model, the dynamics is ultrametric although the statics is not. The issue of finite time FDT plot is also addressed. We discuss multi-level extensions of the trap model and argue

that dynamical ultrametricity should be generic at ‘short times’, i.e. at the beginning of the aging region. We show thermoremanent magnetization data that support this idea. The relation with $1/f$ noise, already discussed in this context [7], is recalled.

II. THE MODEL

The trap model, introduced in the context of aging in [4,5] and further developed in [6], is one of the simplest soluble model exhibiting a dynamical glass transition. In this model, one considers a particle which is trapped in low energy states i of depth $E_i > 0$, where the E_i are random variables distributed according to $\rho(E) = \frac{1}{T_c} e^{-E/T_c}$. The dynamics is chosen to be activated: each particle stays in trap i an exponential random time, equal on average to $\tau_0 e^{E_i/T}$. The quantity τ_0 is a microscopic time scale which we shall take as the time unit in the following. When the particle leaves the trap, it chooses at random a new one among all the others. As a consequence, at high temperature ($T > T_c$), the particle spends most of its time in the small traps, because the number of these traps (the entropic factor) dominates the Boltzmann factor and the system equilibrates. On the contrary, for $T < T_c$, the Boltzmann factor is dominant and the particle explores deeper and deeper traps, so that the system never equilibrates (in the limit of an infinite number of traps). In this regime, the dynamics ages: correlation and response functions are no longer time translation invariant, but depend both on the waiting time t_w and the total time $t_w + t$.

III. CORRELATION FUNCTION

Correlation functions are useful tools to characterize the dynamics and to compare several models. The simplest (but nevertheless non trivial) correlation in the trap model is $\Pi(t_w + t, t_w)$ defined as the probability to remain in the same trap during the time interval $[t_w, t_w + t]$. As we are considering the infinite dimensional (or fully connected) model, this probability is also equal to the probability $P(t_w + t, t_w)$ to be in the same trap at $t_w + t$ as at t_w , since the probability to go back to the same trap vanishes in the limit of an infinite number of traps. (This is not true in finite dimension. For instance, in one dimension, $\Pi(t_w + t, t_w)$ and $P(t_w + t, t_w)$ scale in a different way with t_w).

The correlation function $\Pi(t_w + t, t_w)$ has been calculated in the general case in [7]. We shall now focus on the critical case $T = T_c$, for which we have:

$$\Pi(t_w + t, t_w) = \int_1^{t_w} du \frac{1 - \exp[-(t_w + t - u)]}{(\log u)(t_w + t - u)} \quad (2)$$

for large t_w . We note that this expression simplifies further in the limit $t, t_w \rightarrow \infty$. In particular, the exponential term vanishes since $t_w + t - u > t$. Writing $u = t_w(1 - v)$, we get

$$\Pi(t_w + t, t_w) \simeq \int_0^{1 - \frac{1}{t_w}} \frac{dv}{[\log t_w + \log(1 - v)][v + \frac{t}{t_w}]} \quad (3)$$

$$\simeq \frac{1}{\log t_w} \int_0^{\frac{t_w}{t}} \frac{dz}{z + 1} + \mathcal{O}\left(\frac{1}{(\log t_w)^2}\right) \quad (4)$$

with the new integration variable $z = \frac{t_w}{t}v$. This expression is readily integrated to give:

$$\Pi(t_w + t, t_w) \simeq \frac{\log(1 + \frac{t_w}{t})}{\log t_w} \quad (5)$$

This relation shows that $\Pi(t_w + t, t_w)$ is not a function of $\frac{t}{t_w}$, at variance with the results that hold the whole low temperature phase $T < T_c$ [7]. On the contrary, taking $t \sim at_w^\alpha$, we obtain, in the limit t_w going to $+\infty$:

$$\Pi(t_w + t, t_w) = \mathcal{C}(\alpha) \quad (6)$$

with $\mathcal{C}(\alpha)$ given by:

$$\mathcal{C}(\alpha) = 1 - \alpha \quad (\alpha < 1) \quad (7)$$

$$= 0 \quad (\alpha \geq 1). \quad (8)$$

Note that $\mathcal{C}(\alpha)$ is a monotonous decreasing function of α . The above result is only true in the limit $t \gg \tau_0$, as well as $\log(t_w/\tau_0) \gg 1$. Indeed, all the finite time results reported in this paper should be understood as first order terms in a $1/\log t_w$ expansion.

An important remark is that the correlation function $\Pi(t_w + t, t_w)$ is a function of $\alpha = \log t / \log t_w$ that cannot be written as $h(t_w + t)/h(t_w)$. The latter ratio naturally appears (with an unknown function h) in the aging part of the solution of the dynamical equation corresponding to one step Replica Symmetry Breaking (RSB) mean field spin glass models. However, for full RSB models where dynamical ultrametricity is indeed expected, the correlation function is given by an infinite sum of contributions coming from different times sectors [1,2]:

$$C(t_w + t, t_w) = \sum_i \mathcal{C}_i \left(\frac{h_i(t_w + t)}{h_i(t_w)} \right), \quad (9)$$

where h_i are unknown (monotonous) functions defining the i th time scale, and the \mathcal{C}_i are monotonously decaying to zero for large arguments.

A useful (but up to now unjustified theoretically) form for $h_i(t)$, that allows one to give some flesh to the above formula, is [2]:

$$h_i(u) = \exp \left[\frac{u^{1-\mu_i}}{1-\mu_i} \right], \quad 0 \leq \mu_i \leq 1. \quad (10)$$

It is easy to see that for this choice of h_i , the time scale on which the ratio $h_i(t_w + t)/h_i(t_w)$ varies significantly is precisely $t_w^{\mu_i}$. The choice $\mu = 0$ therefore corresponds to stationary dynamics, whereas $\mu = 1$ gives full aging. Now, take $t = t_w^\alpha$ (with $0 < \alpha < 1$) in Eq. (9) and take the limit $t_w \rightarrow \infty$. All the sectors such that $\mu_i < \alpha$ have relaxed to zero, whereas the sectors corresponding to $\mu_i > \alpha$ have not decayed at all. Introducing a continuum of different values of μ , we find that the correlation function is given by:

$$C(t_w + t_w^\alpha, t_w) = \int_\alpha^1 d\mu \rho(\mu) C_\mu(1), \quad (11)$$

where $\rho(\mu)$ is the ‘density’ of time sectors of order t_w^μ and $C_\mu(1)$ is the initial value of the correlation function in this sector. From this result, one sees that $C(t_w + t, t_w)$ indeed becomes a function of $\alpha = \log t / \log t_w$ in the long time limit. Therefore, interestingly, the superposition of an infinite number of *subaging* contributions defined by (10) naturally leads to a correlation function that depends on $\log t / \log t_w$, for which the dynamical ultrametricity property is explicit. The critical trap behaviour corresponds to a uniform contribution of all time sectors, i.e. $\rho(\mu) C_\mu(1) = 1, \forall \mu$.

IV. DYNAMICAL ULTRAMETRICITY

As recalled in the Introduction, Cugliandolo and Kurchan have defined dynamical ultrametricity for the correlation function C if the following property is true: take three times $t_1 < t_2 < t_3$, then:

$$C(t_3, t_1) = \min(C(t_2, t_1), C(t_3, t_2)), \quad \forall t_2 \in [t_1, t_3]. \quad (12)$$

Let us show now that $\Pi(t_w + t, t_w)$ at the critical temperature is ultrametric in the sense defined hereabove. It will be useful to introduce the following notations:

$$\begin{aligned} \Pi(t_2, t_1) &= C_1 \\ \Pi(t_3, t_2) &= C_2 \\ \Pi(t_3, t_1) &= C_3 \end{aligned} \quad (13)$$

From the monotonicity of correlation functions, the inequality $C_3 \leq \min(C_1, C_2)$ holds in general. We simply have to check that (at least) one of the two correlations C_1 and C_2 is equal to C_3 . In order to take the infinite time limit, we need to specify how t_2 and t_3 scale with t_1 . A natural parametrization is the following:

$$\begin{aligned} t_3 - t_1 &\sim a t_1^\alpha \\ t_2 - t_1 &\sim b t_1^\beta \end{aligned} \quad (14)$$

We can now look at various cases:

- If $\beta < \alpha$, then one has, for large t_1 :

$$t_3 - t_2 \sim a t_1^\alpha - b t_1^\beta \sim a t_1^\alpha \quad (15)$$

so that $C_1 = C(\beta)$ and $C_2 = C_3 = C(\alpha) < C(\beta)$.

- Assuming now $\beta = \alpha$, we get $C_1 = C_3 = C(\alpha)$, as well as $C_2 \geq C_3$.

As a result, we have shown that the relation $C_3 = \min(C_1, C_2)$ always holds, which implies that dynamical ultrametricity is satisfied in this model.

The appearance of dynamical ultrametricity can be considered as a signature of the existence of many time scales involved in the dynamics. This is indeed the case in the present model, even though there is no static ultrametricity to account for this hierarchy of time scales. This property in fact arises naturally from the exact balance, at the critical point, of the Boltzmann weight $e^{E/T}$ and of the entropic factor $\rho(E)$: the probability for the particle to have a given energy (or equivalently, a given trapping time, in logarithmic scale) is essentially uniform on the interval $[0, T_c \log t_w]$. In other words, dynamical ultrametricity here is a consequence of the critical scale invariance.

V. FINITE TIME ANALYSIS

An interesting analysis was introduced by Cugliandolo and Kurchan in the context of the dynamical analysis of the Sherrington-Kirkpatrick (SK) model [1]. These authors made the assumption that, in the limit of large times, there exists a certain function $f(x, y)$, not necessarily smooth, such that $C_3 = f(C_1, C_2)$. We have shown in the previous section that such a function indeed exists in the present case and is given by $f(x, y) = \min(x, y)$.

A useful representation, proposed in [1], is to plot in the (C_1, C_2) -plane the curves of constant $C_3 = f(C_1, C_2)$, which reduces for our case to two straight lines ($C_1 = C_3$ and $C_2 = C_3$) at right angle. For finite times, the function f has to include a time scale as third argument, so that $C_3 = F(C_1, C_2; t_1)$, where $f(x, y) = \lim_{t_1 \rightarrow \infty} F(x, y; t_1)$. The (C_1, C_2) -plane representation is then a good way to visualize the convergence towards the asymptotic function $f(C_1, C_2)$. It has been used with numerical data to test if dynamic ultrametricity holds in realistic systems, with rather inconclusive results [1,8], except in a few cases, like in a recent study of the 4-dimensional Edwards-Anderson model [9].

Let us apply this procedure to the critical trap model. In order to deal with finite time expressions, we shall come back to Eqn. 5, restated as follows:

$$C(t_j, t_i) = \frac{\log(1 + \frac{t_i}{t_j - t_i})}{\log t_i} \quad (16)$$

Inverting these relations so as to express t_2 and t_3 as functions of t_1 , C_1 and C_3 , we can write an explicit expression for $F(C_1, C_2; t_1)$:

$$C_3 = F(C_1, C_2; t_1) = -\frac{1}{\log t_1} \log \left(t_1^{-C_1} + t_1^{-C_2} (1 - t_1^{-C_1})^{1+C_2} \right) \quad (17)$$

In order to plot the constant C_3 curves, it will be useful to express also C_2 as a certain function $G(C_1, C_3; t_1)$:

$$C_2 = G(C_1, C_3; t_1) = \frac{\log(1 - t_1^{-C_1}) - \log(t_1^{-C_3} - t_1^{-C_1})}{\log t_1 - \log(1 - t_1^{-C_1})} \quad (18)$$

The resulting plots in the (C_1, C_2) -plane are displayed in Fig. 1, for $C_3 = 0.1, 0.3, 0.5, 0.7$ and for three different times ($t_1 = 10^5, 10^{10}$ and 10^{15}). Note that the convergence is very slow close to the infinite time singularity.

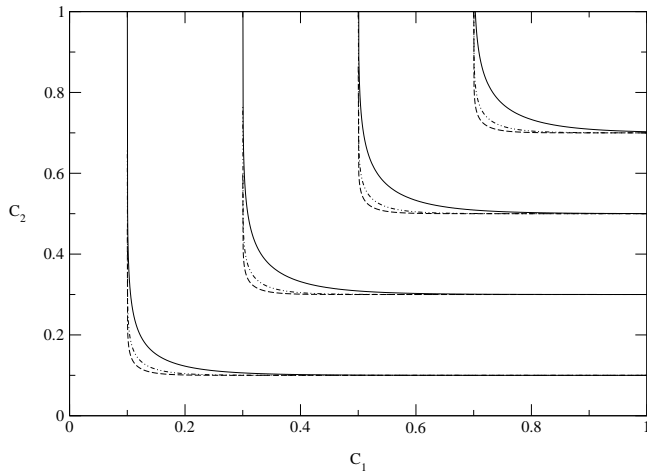


FIG. 1. Plot of constant $C_3 = F(C_1, C_2; t_1)$ in the (C_1, C_2) -plane. From left to right, curves correspond to $C_3 = 0.1, 0.3, 0.5, 0.7$. Each set of three curves shows the convergence with t_1 towards the asymptotic function $f(C_1, C_2) = \min(C_1, C_2)$: $t_1 = 10^5$ (full line), 10^{10} (dot-dashed line), 10^{15} (dashed line).

Cugliandolo and Kurchan also introduced in [1] the notion of correlation time scales through the representation of $f(C, C)$ versus C . As already mentioned before, $f(C, C) \leq C$ in general. There may exist some special ‘fixed’ points C^* such that $f(C^*, C^*) = C^*$. Each of these fixed points has been shown to be associated with a correlation time scale. If dynamical ultrametricity holds in a particular time sector, all C ’s belonging to a certain interval $[C', C'']$ are fixed points. In our case, ultrametricity holds over the full correlation interval $[0, 1]$. But in our model, we can go beyond the infinite time analysis and quantify the convergence of $F(C, C; t_1)$ with t_1 towards the asymptotic function $f(C, C) = C$. We find:

$$F(C, C; t_1) = C - \frac{\log(1 + (1 - t_1^{-C})^{1+C})}{\log t_1} \quad (19)$$

For $C > 0$, this expression simplifies further at large times:

$$F(C, C; t_1) \simeq C - \frac{\log 2}{\log t_1} \quad (20)$$

Interestingly, the leading correction does not depend on C , and it is valid only if C is not too close to 0. Fig. 2 displays the plots of $F(C, C; t_1) - C$, for the same values of t_1 as in Fig. 1.

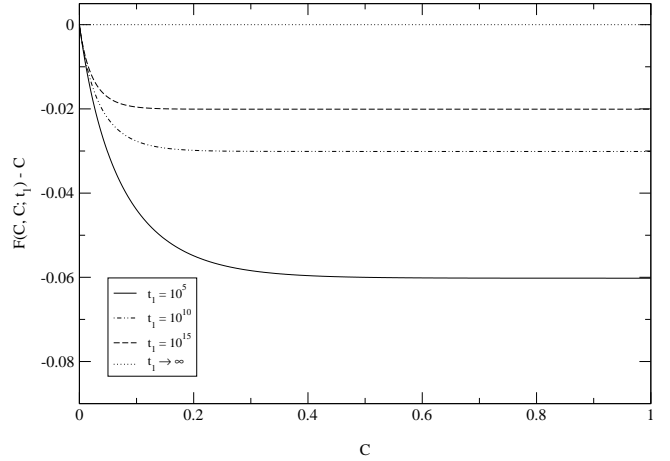


FIG. 2. Plot of $F(C, C; t_1) - C$ versus C for the same values of t_1 as in Fig. 1. The dotted curve, corresponding to the infinite time limit $f(C, C) = C$ is added for comparison. The departure from $f(C, C) = C$ is almost independent of C , except for C close to 0.

This last result may also be interpreted in the framework of Fig. 1. For a given value of C_3 , the point $C(t_1)$ defined by the relation $F(C(t_1), C(t_1); t_1) = C_3$ converges to the infinite time right angle singularity $C_1 = C_2 = C_3$ as (see Eqn. 20):

$$C(t_1) \simeq C_3 + \frac{\log 2}{\log t_1} \quad (21)$$

This logarithmic correction may explain why it seems so difficult to observe the convergence towards the ultrametric relation in experimental or numerical data, where only a few decades (usually between four and six) are available.

VI. THE EFFECT OF ‘SHEAR’

The effect of an external ‘shear’ (or power injection) on aging was investigated in the context of mean field models in [8], and in the context of the trap model in [10]. In both models, aging is interrupted by the shear

beyond a time scale τ_r which diverges as the shear rate $\dot{\gamma}$ tends to zero. In the model considered in [10], this time scale is given by:

$$\tau_r \simeq \frac{1}{\dot{\gamma}} \left(\log \frac{1}{\dot{\gamma}} \right)^{\frac{1}{2}}. \quad (22)$$

For $\tau \ll \tau_r$, the (power-law) distribution of trapping times is in a first approximation unaffected by the shear, whereas for longer times, the distribution decays exponentially. In the limit where the waiting time t_w is much larger than τ_r , the dynamics of the model becomes stationary and one finds, for $t \ll \tau_r$, the same result as above with t_w replaced by τ_r :

$$C(t + t_w, t_w) = C \left(\frac{\log t}{\log \tau_r} \right) \simeq 1 - \frac{\log t}{\log \tau_r}. \quad (23)$$

As discussed in [8,9], dynamical ultrametricity in this context manifests itself by the appearance of an infinity of time scales in the limit $\tau_r \rightarrow \infty$ (i.e. $\dot{\gamma} \rightarrow 0$): the time needed for the correlation to decay to a certain value c diverges as τ_r^{1-c} (see [11] for a further discussion).

VII. THE FLUCTUATION-DISSIPATION RATIO

It is interesting to study the fluctuation dissipation ratio X in the trap model at the critical point. This ratio is defined as:

$$X(t_2, t_1) = \frac{TR(t_2, t_1)}{\partial C(t_2, t_1)/\partial t_1}, \quad (24)$$

where $R(t_2, t_1)$ is the response of the system at time $t_2 > t_1$ to a small bias field applied at time t_1 (see [7,12] for details). In the trap model, where the field only changes the trapping time of the starting site, one finds that the following relation holds in general:

$$TR(t_2, t_1) = -\frac{\partial C(t_2, t_1)}{\partial t_2}. \quad (25)$$

Using this result, one sees that in the ‘liquid’ phase $T > T_c$ where all two time functions only depend on time differences, $X \equiv 1$: the usual Fluctuation-Dissipation Theorem (FDT) holds. In the glass phase, on the other hand, one finds that $X(t_2, t_1) = t_1/t_2$: the value of X is non trivial in the whole scaling regime $t_2 - t_1 \sim t_1$. Right at the critical point $T = T_c$, one can express X as:

$$X(t_2, t_1) = \frac{(t_1^C - 1)^2}{t_1^C(t_1^C + 1 - C)} \quad C \equiv C(t_2, t_1) > 0. \quad (26)$$

So for any fixed $C > 0$, X tends to 1 in the asymptotic limit $t_1 \rightarrow \infty$. We show in Fig. 3 the now famous plot of the integrated response versus C , which should yield a straight line of slope $-1/T_c$ when the FDT holds. Again,

one sees a very slow convergence towards the asymptotic result for small values of C .

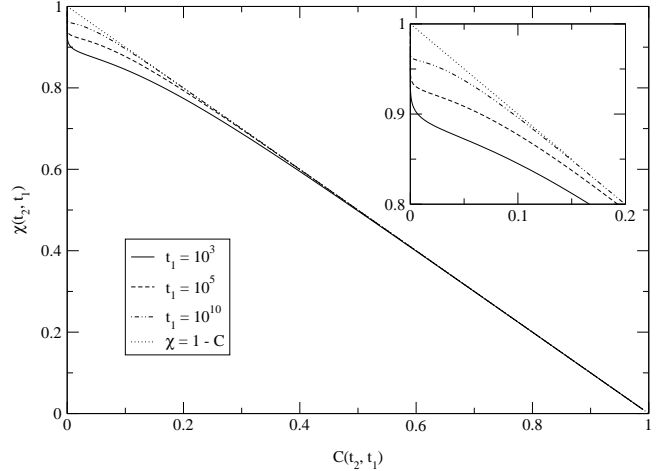


FIG. 3. Plot of integrated response $\chi(t_2, t_1)$ versus $C(t_2, t_1)$ parametrized by t_2 , for $t_1 = 10^3, 10^5$ and 10^{10} from bottom to top. The local slope is equal to $-X(t_2, t_1)/T_c$, and it converges very slowly towards the asymptotic value $-1/T_c$ for small C (see inset). T_c is chosen as the temperature unit.

VIII. THE MULTILEVEL TRAP MODEL AND DISCUSSION

The simple trap model described above can be considered as a one-step RSB model. In order to generalize the model to a full RSB one, it has been proposed in [7] (and further studied in [13]) to follow Parisi’s procedure for the static solution of the SK model. Roughly speaking, it means that each trap is recursively subdivided into a new series of traps, in a hierarchical manner. Each level k of traps is characterized a certain overlap between states q_k and by an exponential probability distribution of the energy barriers, with a critical temperature T_c^k depending on the level index k . The critical temperatures are related to Parisi’s function $x_k = x(q_k)$ as $T_c^k = T/x_k$, and satisfy the relations $T_c^k < T_c^{k-1}$. At any temperatures, the levels of the tree corresponding to $q > q_{EA}(T)$, where q_{EA} is the Edwards-Anderson parameter, are such that $x_k > 1$, so that these levels are equilibrated (i.e. $T_c^k < T$).

In the single level trap model, the correlation function $\Pi(t_w + t, t_w)$ in the aging phase $T < T_c$ behaves at short times as:

$$\Pi(t_w + t, t_w) \simeq 1 - \frac{\sin \pi x}{\pi(1-x)} \left(\frac{t}{t_w} \right)^{1-x} \quad 1 \ll t \ll t_w \quad (27)$$

with $x = \frac{T}{T_c}$. In the multi-level model with a finite number M of levels, the total correlation function is defined as:

$$\begin{aligned}
C(t_w + t, t_w) &= \sum_{k=0}^M q_k [\Pi_k(t_w + t, t_w) - \Pi_{k+1}(t_w + t, t_w)] \\
&= q_0 + \sum_{k=1}^M (q_k - q_{k-1}) \Pi_k(t_w + t, t_w) \quad (28)
\end{aligned}$$

q_k being the k^{th} level overlap, and $\Pi_k(t_w + t, t_w)$ is the probability that the process has never jumped beyond the k^{th} layer of the tree between t_w and $t_w + t$ with the convention that $\Pi_0(t_w + t, t_w) = 1$ and $\Pi_{M+1}(t_w + t, t_w) = 0$ (see [7] for details). From this definition, we see that $C(t_w + t, t_w)$ is dominated at short times by the levels k with x_k close to 1, for which the short time singularity is strongest. (We assume that $T < T_c^0$, i.e. that at least one level is aging). Therefore, we expect to observe the dynamical ultrametricity associated to the level k^* for which $x_{k^*} = 1$ in the ‘short’ time regime $\log t / \log t_w < 1$, before the t/t_w regime associated to the levels $k < k^*$ sets in. (Note that if $C(t_w + t, t_w)$ is a function of t/t_w , then the function $f(x, y)$ defined above cannot be equal to $\min(x, y)$).

Interestingly, ‘short time’ dynamical ultrametricity exists for the hierarchical tree model in the whole low temperature phase, but has no relation with the static ultrametricity built in the tree structure which encodes Parisi’s RSB solution. Thus the origin of dynamical ultrametricity in the generalized trap model is again very different from the dynamical ultrametricity found in mean field spin glass models, which in the latter case is deeply related to the Parisi function $x(q)$ which encodes the structure of the tree. As mentioned in the introduction, the physical interpretation of aging in the two pictures are radically different, although some of the phenomenology is very similar.

Aging experiments in spin-glasses have been interpreted within the framework of the multi-level trap model in [7]. The need for several levels comes from rejuvenation and memory in temperature shift experiments, but also from the detailed shape of the thermo-remanent magnetization (TRM) relaxation at a given temperature, which shows that the short time and long time singularities are described by different exponents x_k [14]. It is natural to interpret the different levels of the hierarchy in terms of length scales, as is actually suggested by the RSB solution of the pinned manifold problem [15]. Therefore, one can expect that for any temperature within the spin-glass phase, there will be a particular ‘critical’ length scale ℓ for which $x = x(\ell) = 1$, that will contribute to the correlation and response functions as a function of $\log t / \log t_w$. We have reanalyzed some TRM data in the spirit of the present discussion. We show in Fig. (4) the decay of the TRM in AgMn at $T = 0.75T_g$ [16], plotted as a function of $s(t, t_w) = \log(1 + t/t_w) / \log t_w$, as suggested by Eq. (5). This figure shows that the rescaling is very good at short times, but is violated for large t/t_w (corresponding to small $s(t, t_w)$). From Eq. (28), we indeed

expect to observe the sum of a contribution $\mathcal{M}(k < k^*)$ from the levels $k < k^*$ (that only depends on t/t_w) and a contribution from $k \simeq k^*$, proportional to $s(t, t_w)$. When $\alpha = \log t / \log t_w < 1$, the first contribution does not vary much, since $(t/t_w)^{1-x_k} = t_w^{(\alpha-1)(1-x_k)}$ is very small for large t_w , whereas the second contribution is a function of α . This suggests that in the short time regime one should observe:

$$M(t_w + t, t_w) = \mathcal{M}(k < k^*) + m_1 s(t, t_w), \quad (29)$$

where m_1 is a certain prefactor. The quantity $\varphi = m_1 / (\mathcal{M}(k < k^*) + m_1)$ measures the relative contribution of the levels $< k^*$ and around k^* in the total decay of the signal. The dashed line shown in Fig. 4 is a linear fit of the initial decay, as a function of s , from which one extracts (in this particular example) $\varphi \approx 0.2$.

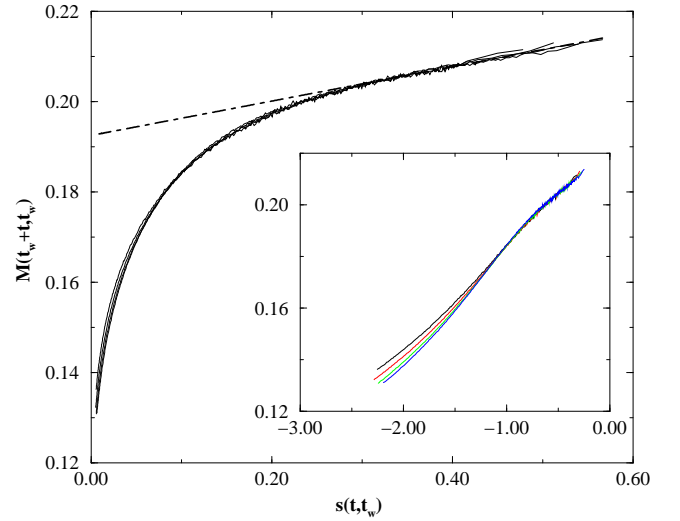


FIG. 4. Plot of TRM data in a AgMn spin-glass at $T/T_g = 0.75$, for $t_w = 300, 1000, 3000$ and 10000 seconds. The horizontal axis is the variable $s(t, t_w)$ defined in the text. The rescaling is very good for $s(t, t_w) > 0.3$ approximately (or $\log t / \log t_w < 0.7$), but becomes inadequate for longer times t , as more clearly seen in the inset where $\log s$ is used. The dashed line is an affine fit of the short time part of the data.

In all the above formulas, time is implicitly measured in units of a microscopic time τ_0 . The value of τ_0 is not necessarily an individual flip time $\sim 10^{-12}$ sec., since collective dynamics may exist in the vicinity of the transition point (see [17,18] for a detailed discussion). In Fig. 4, we have chosen $\tau_0 = 10^{-5}$ sec. to achieve the best rescaling. This value is very close to the one extracted from the analysis of [17].

The conclusion is that the TRM data is indeed compatible with a $\log t / \log t_w$ behaviour for short times. Note that this behaviour is tantamount to a logarithmic dependence of the a.c. susceptibility, or else to $1/f$ noise. We have insisted in previous papers [7,13] on the fact that the

existence of levels in the vicinity of $x = 1$ generically leads to $1/f$ noise for long times and low frequencies, because the contribution of other levels (both above and below k^*) become negligible in that regime. We show here that the degrees of freedom contributing to $1/f$ noise also give rise to dynamical ultrametricity.

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